Corrections to Richard Lyons DSPrelated Blog: *FFT Interpolation Based on FFT Samples: A Detective Story With a Surprise Ending* - April, 2018.

Derek Rowell - April 22, 2018

The equation (B-4) in Appendix B of the downloadable pdf version of the article

$$\sum_{m=0}^{N-1}(e^{j2\pi m/N}z^{-1})^N = \frac{1-(e^{j2\pi m/N}z^{-1})^N}{1-e^{j2\pi m/N}z^{-1}} = \frac{1-z^{-N}}{1-e^{j2\pi m/N}z^{-1}} \tag{1}$$

has an easily seen arithmetic error in the second step, and should be:

$$\sum_{m=0}^{N-1}(e^{j2\pi m/N}z^{-1})^N = \frac{1-(e^{j2\pi m/N}z^{-1})^N}{1-e^{j2\pi m/N}z^{-1}} = \frac{1-e^{j2\pi m}z^{-N}}{1-e^{j2\pi m/N}z^{-1}}. \tag{2}$$

Unfortunately, this error permeates down through all subsequent results in Appendices B and C, as well as throughout the main text, rendering all presented results incorrect. The result is that while the spectral magnitudes reported are correct, the phases are in error.

Using Lyon's nomenclature, corrections to Eqs. (B-6), (C-2) , and Eq. (3) in the main text become

$$X(k) = X(z)|_{z=e^{j2\pi k/N}} = \frac{1}{N}\sum_{m=0}^{N-1}X(m)\frac{1-e^{j2\pi(m-k)}}{1-e^{j2\pi(m-k)/N}} \tag{3}$$

where k is non-integer.

The corrected version of Eq. (B-9) and Eq. (2) in the main text is:

$$X(k) = \frac{1}{N}\sum_{m=0}^{N-1}X(m).e^{-j\pi(k-m)(1-1/N)}\frac{\sin(\pi(k-m))}{\sin(\pi(k-m)/N)}, \tag{4}$$

which is of the form

$$X(k) = \sum_{m=0}^{N-1}X(m)P(k-m) \tag{5}$$

where

$$P(k) = \frac{1}{N}e^{-j\pi(k)(1-1/N)}\frac{\sin(\pi k)}{\sin(\pi k/N)}$$

is the periodic interpolation formula. Equation (5) is in agreement with the solution published in Proakis and Manolakis (albeit with a different nomenclature). [1]

[1] J. Proakis and D. Manolakis, *Digital Signal Processing-Principles, Algorithms, and Applications*, Prentice Hall, Upper Saddle River, New Jersey, (3rd Ed., p 397), 4th Ed., pp 452-453