Correcting I-Q Imbalance in Direct Conversion Receivers

S.W. Ellingson*

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1 Introduction

Analog direct conversion receivers are notorious for I-Q imbalance. Consider the conversion of a single tone at RF to baseband. Ideally, the I and Q outputs of the receiver are

$$I(t) = \cos(\omega t) \text{ and} \tag{1}$$

$$Q(t) = \sin(\omega t) \ , \tag{2}$$

respectively. ω is the baseband frequency of the tone. With no loss of generality, we have normalized the magnitude to unity and the phase to zero, as these quantities are not relevant to this discussion. In contrast, a realistic direct conversion receiver produces:

$$I'(t) = \alpha \cos(\omega t) + \beta_I \tag{3}$$

$$Q'(t) = \sin(\omega t + \psi) + \beta_Q \tag{4}$$

where ψ is the phase error, which we have assigned to the "Q" path, α is the magnitude error, which we have assigned to the "I" path, and β_I and β_Q are the DC biases associated with each path. The allocation of error mechanisms to paths is completely arbitrary and implies no loss of generality.

*ElectroScience Laboratory, The Ohio State University, 1320 Kinnear Rd., Columbus, OH 43212; Voice: (614) 292-7981, Fax: (614) 292-7297, E-mail: ellingson.1@osu.edu.

2 Theory

Correcting β_I and β_Q is very simple. For example, β_I is simply the mean of $I'(t)$ over an integer number of periods. Given this estimate, the correction is simply a matter of subtracting β_I from the "I" path signal. The process is the same for the "Q" path. Then, we are left with:

$$I''(t) = \alpha \cos(\omega t) \tag{5}$$

$$Q''(t) = \sin(\omega t + \psi) \tag{6}$$

This can be rewritten in matrix form as:

$$\begin{bmatrix} I''(t) \\ Q''(t) \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ \sin(\psi) & \cos(\psi) \end{bmatrix} \begin{bmatrix} I(t) \\ Q(t) \end{bmatrix} \tag{7}$$

where we have used the identity:

$$\sin(wt + \psi) = \sin(\omega t)\cos(\psi) + cos(\omega t)\sin(\psi) \tag{8}$$

Thus, we find that the correction is:

$$\begin{bmatrix} I(t) \\ Q(t) \end{bmatrix} = \begin{bmatrix} \alpha^{-1} & 0 \\ \alpha^{-1}\tan(\psi) & \sec(\psi) \end{bmatrix} \begin{bmatrix} I''(t) \\ Q''(t) \end{bmatrix} \tag{9}$$

Thus, it remains only to find α and ψ. To find α, let us first define:

$$< x(t) > \; = \; \frac{1}{NT}\int_{t-NT}^{t} x(u)\; du \tag{10}$$

where T is the period $2\pi/\omega$ and N is any integer greater than zero. Note that

$$< I''(t)I''(t) >= \alpha^2 < \cos^2(\omega t) >= \alpha^2 \left\langle \frac{1}{2} + \frac{1}{2}\cos(2\omega t) \right\rangle = \frac{1}{2}\alpha^2 \tag{11}$$

and by similar analysis

$$< I''(t)Q''(t) >= \frac{1}{2}\alpha^2 \sin(\psi)\ . \tag{12}$$

Thus, Equation 11 can be used to find α, and then Equation 12 can be used to find $\sin(\psi)$. Assuming $|\psi| < \pi/2$ (hopefully any phase imbalance would be much smaller than this!), we can obtain $\cos(\psi)$ directly from $\sin(\psi)$.

3 Summary

Here's the complete algorithm for identifying the I-Q imbalance and correcting it:

1. Apply a tone that appears at baseband frequency ω. Note the measurement and correction will only be valid at this frequency; however, these parameters typically vary slowly with frequency.

2. Compute $\beta_I =< I'(q) >$ and $\beta_Q =< Q'(t) >$.

3. Compute $I''(t) = I'(t) - \beta_I$ and $Q''(t) = Q'(t) - \beta_Q$.

4. Compute $\alpha = \sqrt{2 < I''(t)I''(t) >}$.

5. Compute $\sin(\psi) = (2/\alpha) < I''(t)Q''(t) >$

6. Compute $\cos(\psi) = \sqrt{1 - \sin^2(\psi)}$.

7. Compute the correction matrix parameters:
   $A = 1/\alpha$
   $C = -\sin(\psi)/(\alpha\cos(\psi))$
   $D = 1/\cos(\psi)$

8. The correction can now be applied to subsequent data as follows:

$$\begin{bmatrix} I(t) \\ Q(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ C & D \end{bmatrix} \begin{bmatrix} I'(t) - \beta_I \\ Q'(t) - \beta_Q \end{bmatrix} \tag{13}$$